System for controlling the vulcanisation characteristics of a rubber mix

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Summary

The present authors [1–3] have shown the possibility of controlling the vulcanisation characteristics of a rubber mix by adjusting the mass doses of sulphur and sulphenamide, and have constructed a mathematical model of the controlled entity in respect of the control actions. In the present work, a linear, quadratic, stochastic controller of vulcanisation characteristics is synthesised, and, by computer modelling, under real production disturbances, its effectiveness is investigated.

Expanded Mathematical Model of the Controlled Entity

To allow for the statistical characteristics of disturbances during synthesis of the controller, an expanded mathematical model of vulcanisation characteristic control is constructed, including, besides the controlled entity, disturbance shaping filters. A block diagram of the model is presented in Figure 1. The expanded model contains elements with transfer functions:

- \( W_1(p) = b_{11} e^{\omega t} = 0.095 e^{\omega t} \)
  is the transfer function along the channel of influence of the mass dose of sulphur on the maximum shear moment;
- \( W_2(p) = b_{21} e^{\omega t} = 0.068 e^{\omega t} \)
  is the transfer function along the channel of influence of the mass dose of sulphenamide on the time constant of rubber mix vulcanisation;
- \( W_3(p) = b_{12} e^{\omega t} = 0.027 e^{\omega t} \)
  is the transfer function along the channel of influence of the mass dose of sulphenamide on the maximum shear moment;
- \( W_4(p) = b_{22} e^{\omega t} = 0.025 e^{\omega t} \)
  is the transfer function along the channel of influence of the mass dose of sulphenamide on the time constant of rubber mix vulcanisation.

where \( \tau = 1 \) is the transport time lag of the control actions, because, with the adopted control scheme, the result of analysis of the controlled parameters makes it possible to adjust the dose of the initial ingredients only for the next shift, and \( p \) is the Laplace operator.

Figure 1. Block diagram of the control of rubber mix reactivity in the presence of disturbances in the production process: \( u_1, u_2 \) – control action along the channel of the dose volume of sulphur and sulphenamide respectively; \( y_1, y_2 \) – controlled parameters of the entity – maximum moment and rate constant respectively; \( W_1(p), W_2(p), W_3(p), W_4(p) \) – transfer functions; \( W_{f1}(p), W_{f2}(p) \) – shaping filters for disturbance generation; \( w_1(t), w_2(t) \) – random processes; \( f_1(t), f_2(t) \) – random disturbances
$W_1(p)$ and $W_2(p)$ are the transfer functions of the shaping filters for the generation of disturbances with respect to the maximum shear moment and the time constant of rubber mix vulcanisation $f_1(t)$ and $f_2(t)$ with characteristics taking place in production on change in the characteristics of the initial ingredients. To the inputs of the shaping filters are fed random processes $w_1(t)$ and $w_2(t)$ of the ‘white noise’ type, ensuring the production of disturbances $f_1(t)$ and $f_2(t)$ with specified variances.

On the basis of the spectral densities of the disturbances [2] and known methods of construction of shaping filters [4, 5], the following transfer functions were obtained:

$$
W_1(p) = \frac{292p + 5.548}{1000p^2 + 104p + 3.86}, \quad W_2(p) = \frac{0.6}{7.143p + 1}
$$

(1)

By transformation of the expanded model of the controlled entity in the Matlab system [6–12], an expanded model in state space was obtained:

$$
egin{cases}
    x[k + 1] = Ax[k] + Bu[k], \\
y[k] = Cx[k] + Du[k]
\end{cases}
$$

(2)

where $x$ is the column vector of the parameters of state of an entity of dimensions $n = 7$, $u$ is the vector of input actions, including control actions of dimensions $r = 2$ ($u_1$, $u_2$) and a random process of the ‘white noise’ type of dimensions $p = 2(w_1, w_2)$ for the shaping of disturbances, and $y$ is the vector of controlled variables of dimensions $k = 2$.

The first equation in system (2) is the equation of state describing the connection of the parameters of state of the process with the control actions. The second equation in system (2) describes the connection of the measured process parameters with the parameters of state and control actions and is the equation of observation. Matrices $A$, $B$, $C$, and $D$ describe the dynamic properties of the given controlled entity. For the controlled entity under examination, the matrices of state are equal to [6]:

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.9 & -0.45 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0.86 \\
\end{bmatrix},
$$

$B = \begin{bmatrix}
0.095 & 0 & 0 & 0 \\
0 & 0.027 & 0 & 0 \\
-0.007 & 0 & 0 & 0 \\
0 & -0.025 & 0 & 0 \\
0 & 0 & 0.50 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.25 \\
\end{bmatrix}
$$

$C = \begin{bmatrix}
1 & 1 & 0 & 0 & 0.56 & -0.27 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0.31 \\
\end{bmatrix}
$$

$D = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
$}

**SYNTHESIS OF THE OPTIMUM CONTROLLER [6–11]**

The optimum stochastic controller contains a linear, optimum, determined controller and an observing device based on a Kalman filter for assessing the nominal mean vector of state of the system with specified values of the control actions and controlled variables. The problem of synthesising the optimum controller consists in finding a controller that ensures a minimum value of the quadratic quality criterion for stochastic discrete controlled entity (2):

$$
J(u) = E \sum_{k=1}^{N} \left( Cx[k]^T R \left[ Cx[k] + u[k]^T R_2 u[k] \right] \right) \rightarrow \min
$$

(3)

where $E$ is the mathematical expectation operator, $C = Cx[k]^T R Cx[k]$ is the criterion element describing the contribution of the controlled parameters of the entity with a diagonal matrix $R$ of weight coefficients making it possible to single out the most important controlled parameters, and $u[k]^T R_2 u[k]$ is the criterion element taking into account the costs of control and ensuring allowance for constraints with respect to the control actions.

For discrete controlled entity (2), the optimum controller ensuring the minimum value of the quadratic criterion (3) is the linear controller:

$$
u = -K \cdot \dot{x}
$$

(4)

the optimum coefficient matrix of which is defined by the expression:

$$
K = (R_2 + B^T (C^T R C + P) B)^{-1} B^T (C^T R C + P) A
$$

(5)

where $P$ is a symmetrical positive-definite $n \times n$ matrix defined by the Riccati equation:
\[ P = R_1 + A^T P A - A^T P B R_2 B^T P A^{-1} B^T A \]

(6)

The observing device, based on a Kalman filter and ensuring filtration and estimation of unmeasured coordinates of the state of the entity, is described by the expression:

\[ \dot{x}(k+1) = A x(k) + B u(k) + F (y(k) - C x(k)) - D x(k) \]

(7)

where \( F \) is the matrix of internal feedback of the Kalman filter, defined by the expression:

\[ F = A S C^T (Q_\beta + C S C^T)^{-1} \]

(8)

where \( S \) is a symmetrical positive-definite matrix defined by the Riccati equation:

\[ S = A S A^{-1} - A S C^T (Q_\beta + C S C^T)^{-1} C S A^T + Q_\alpha \]

(9)

where \( Q_\alpha \) and \( Q_\beta \) are covariational matrices of white noise of the disturbances and errors of observations.

The optimum linear regulator and the Kalman observer were constructed in Control System Matlab [11]. In the process of synthesis, by appropriate selection of the elements of matrix \( R_1 \), control of the output variables of the entity was guaranteed, and, by appropriate selection of the ratios of matrices \( R_1 \) and \( R_2 \), control actions within the ranges of variation allowed by the production process were ensured. Thereby, a controller is constructed that, with specified control action constraints, ensures minimum variances of the controlled variables. As a result, the following matrix of the controller is obtained:

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 5.82 \\
0 & 0 & 0 & -1.53 & 0.75 \\
2.86 & 3.28 & 0.75 & -11.6
\end{pmatrix}
\]

The full block diagram of the developed control system is presented in Figure 2. The diagram shows the controlled entity and the multidimensional quadratic stochastic controller, including the observing device and the linear optimum controller.

Analysis of the effectiveness of the constructed control system was conducted by mathematical modelling. Here, on a computer, the work of the controller in feeding real disturbances for the production process was modelled. Figure 3 gives times series of controlled variables without the controller (open system) and with the controller in operation (closed system).

The results of modelling showed that the constructed controller makes it possible to reduce output parameter variance. The sum of squares of the deviations of the maximum moment with the controller decreases by a factor of 1.88, and the vulcanisation time by a factor of...
of 1.72, which is a high index for the process control system.

Thus, a control system has been developed that makes it possible, by adjusting the doses of sulphur and sulphenamide, to reduce the influence of change in the characteristics of the initial ingredients on the characteristics of the rubber mix and the vulcanisation process, and to increase the stability of the mechanical characteristics of the rubber mechanical goods produced.

REFERENCES


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