Determining the parameters of a viscoelastic mathematical model of the behaviour of rubber compounds for the purpose of engineering analysis in computer program packages

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Investigations have shown [1] that, when solving thermomechanical problems in the LS-DYNA program package, the best agreement between actual and virtual tests of rubber compounds under up to 50% strain is obtained when the viscoelastic mathematical model MAT_VISCOELASTIC is used.

The expediency of using a less complex or more complex model is determined by the nature and magnitude of deformation of the rubber, as the requirements laid down should be minimal and adequate. The elastic and viscous properties of material can be characterised by the corresponding physicomechanical properties (equilibrium and non-equilibrium elastic and shear moduli, Poisson's ratio), and also by the relaxation curves. In some cases, with account taken of the temperature fields in the rubber, it is also necessary to consider the temperature dependences of the physicomechanical properties.

It must be pointed out that the viscoelastic model (MAT_VISCOELASTIC) employed in the LS-DYNA program package does not incorporate allowance for the effect of temperature on the parameters of the model. When such allowance is necessary, the MAT_VISCOELASTIC_THERMAL model is used. In this work, investigations have been made of the degree of influence of the test regimes on the parameters determined using the MAT_VISCOELASTIC model, including temperature, magnitude of strain, rate, and preloading history.

The initial materials for the studies were specimens of rubbers of different formulation and designation. The following parameters were determined: the tensile elastic modulus \( E \), the bulk modulus \( K \), Poisson's ratio \( \mu \), the load curve (deformation curve), and the stress relaxation curve.

To obtain the given information, it is necessary to conduct tests on rubbers that make it possible to obtain relaxation curves under tension and stress–strain curves under bulk compression.

The relaxation and deformation curves are the base curves for obtaining the most complete information on the viscoelastic properties of rubbers. Tests were conducted on Instron Tensometer 2020 equipment, which made it possible to model different strain regimes of rubber in a wide temperature range (from \(-70°C\) (using a cryochamber) to \(+300°C\)). The shape of the test specimens was in accordance with the GOST 270-75 standard.

The first stage of the work was to determine the elastic modulus of the rubbers. The most complete information on the viscoelastic properties of materials in the rubbery (high-elastic) state is provided by deformation and relaxation curves obtained at different temperatures or different loading rates (Figure 1), and also the time dependence of the elastic modulus \( E(t) \) and the temperature dependence of the elastic modulus \( E(T) \). The elastic modulus must be determined under conditions most approximating actual conditions, i.e. in the region of working rates and strains. It is recommended that it be determined from the tangent to the working region of the deformation curve.
The use of the viscoelastic model is possible in a wide temperature range. According to the principle of temperature–time superimposition [2], an increase in temperature is equivalent to a reduction in frequency or an increase in time of action. That is, the results of tests of rubber compounds at a certain deformation rate and at temperatures \( T_1, T_2, T_3, \) and \( T_4 \) \( (T_1 < T_2 < T_3 < T_4) \) can be reproduced at the same temperature but at different deformation rates \( v_1, v_2, v_3, \) and \( v_4 \) \( (v_1 > v_2 > v_3 > v_4) \) (Figures 1a and b). Here, the \( \sigma_t \) curve is the approximated curve of tensile stress.

Substantiating the choice of the test rate of specimens when determining the parameters of mathematical models in finite element analysis packages reduces to determining the true stress–strain state of mechanical rubber goods under prescribed load and strain regime.

With the aim of establishing the degree of influence of the deformation rate on the elastic modulus, tests were conducted at elongation rates of 50, 100, 300, 500, and 1000 mm/min. As follows from the data obtained, with increase in the test rate by a factor of 20, the position of the curves changes, but negligibly. These changes depend on the type of rubber compounds, namely on their viscoelastic properties.

If, for certain rubbers the rate has a considerable influence, and it is not possible to ensure the required strain regime in view of limitations of the equipment, the elastic modulus can be adjusted using the principle of temperature–time superimposition by conducting tests at different temperatures or finding the dependence of the modulus on the test rate.

When determining the elastic modulus for the viscoelastic model, special attention must be paid to the selection of the magnitude of strain. Primarily, this concerns tensile tests, as the deformation curve is non-linear. Three sections can be singled out on the curve (Figure 2) [3]. Above all, if corresponding strains do not occur in the part under design, the effect of orientation of macromolecules in the direction of strain, which is observed on section III, must be ruled out. In compression tests, the magnitude of strain is lower and therefore has no significant effect on the magnitude of the elastic modulus. For most articles, the magnitude of strain of the rubber compounds does not exceed 80–100%. Tests in the region of such strains again make it possible to exclude or reduce considerably the effect of orientation of macromolecules in the direction of strain.

Investigations were conducted on the influence of the tensile loading regime on the damping factor \( \beta \) during subsequent relaxation. A typical curve taken on the Instron Tensometer 2020 was given in a previous paper [1]. As follows from the data obtained during testing of rubber compounds based on natural rubber, the damping factor \( \beta \) is influenced considerably both by the magnitude and by the rate of strain. Values of \( \beta \) after approximation of the relaxation curves are presented in Table 1.

From Table 1 it can be seen that the damping factor \( \beta \) with increase in strain from 50 to 110% increases by a factor of more than 2.5. Thus, tests for determining \( \beta \) must be conducted with account taken of actual magnitudes of strain. The damping factor can be adjusted by taking

![Figure 1. Typical stress \( \sigma \)-strain \( \varepsilon \) curves (deformation curves) of elastomers: (a) at constant rate \( v = \text{const} \); (b) at constant temperature \( T = \text{const} \)](image)

![Figure 2. Typical deformation curves for elastomers](image)

<table>
<thead>
<tr>
<th>( \varepsilon ) (%)</th>
<th>( \beta )</th>
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<tbody>
<tr>
<td>50</td>
<td>0.0074</td>
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<tr>
<td>62</td>
<td>0.0086</td>
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<tr>
<td>74</td>
<td>0.0114</td>
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<td>82</td>
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<td>92</td>
<td>0.0171</td>
</tr>
<tr>
<td>110</td>
<td>0.0197</td>
</tr>
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</table>

Table 1. Values of the damping factor \( \beta \) under different strains \( \varepsilon \) and with a deformation rate of 500 mm/min
into account its dependence on the magnitude of strain.

The chosen relaxation test time should be minimal but adequate for the damping factor $\beta$ hardly to change during more prolonged relaxation. Note that, for articles operating in a dynamic regime, the duration of relaxation processes is limited, which eliminates the need to conduct prolonged relaxation. Such prolonged processes are necessary only in the case of determining the parameters for rubber compounds operating under static conditions or in a steady-state regime. In this case, the static elastic modulus can be determined.

Thus, the parameters of the rubber compounds must be determined in regimes similar to the service conditions. Of great importance in this case are the rate at which the test is carried out, the form of strain, the magnitude of strain, and especially the prehistory of loading and temperature. Otherwise the parameters obtained will not make it possible to produce an adequate computer model.

An investigation was also made of the influence of the rate of loading and the pressure of compression on the bulk modulus.

The bulk modulus is associated with Poisson’s ratio ($\mu$). It is customary to assume that Poisson’s ratio for rubber compounds is close to 0.5. Under actual service conditions, rubber compounds are practically incompressible materials not because they are actually incompressible but because the tensile elastic modulus is low by comparison with the bulk modulus.

Whereas obtaining deformation and relaxation curves of specific rubbers does not present any great difficulties, obtaining data on the bulk modulus ($K$) is associated with great difficulties. Therefore, for rubber compounds of different grades based on natural rubber, measurements were made of the dependence of the stress under bulk compression on the strain rate and the magnitude of the applied force.

A schematic of the tests is given in Figure 3. The working cell comprises a thick-walled steel cylinder with a well-ground plunger. Tests were conducted using the Instron Tensometer 2020. Here, the accuracy of measuring the force and movement of the plunger was 0.5%. Experiments were conducted at rates of 1–100 mm/min, with a developed pressure on the specimen of up to 16 MPa. The relative error of measurements of $K$ did not exceed 1.9% ($P = 0.95$).

The bulk modulus characterises the compressibility of the material. It is related to Poisson’s ratio $\mu$:

$$K = \frac{E}{3(1-2\mu)}$$

where $K$ is the bulk modulus, $E$ is the elastic modulus, and $\mu$ is Poisson’s ratio.

Hence, Poisson’s ratio can be calculated in the following way:

$$\mu = \frac{1}{2}\left(1 - \frac{E}{3K}\right)$$

In models of material, use is also made of the shear modulus ($G$), which can be either determined or calculated on the basis of the elastic modulus and Poisson’s ratio:

$$G = \frac{E}{2(1+\mu)}$$

To calculate the main physicomechanical properties/parameters of the model, classic relationships were used [4]. Table 2 presents the values of the parameters of the model for three specimens of rubber compounds based on natural rubber of different formulation.

Thus, in a previous paper [1], on the basis of an analysis of models of the behaviour of material in the LS-DYNA program package, for description of the

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$K$ (MPa)</th>
<th>$\mu$</th>
<th>$\beta$</th>
<th>$G_0$ (MPa)</th>
<th>$G_1$ (MPa)</th>
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<tbody>
<tr>
<td>Specimen 1</td>
<td>3.73</td>
<td>346.8</td>
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<td>0.0133</td>
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<td>0.0245</td>
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<tr>
<td>Specimen 3</td>
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<td>343.2</td>
<td>0.4989</td>
<td>0.0128</td>
<td>0.74</td>
<td>0.42</td>
</tr>
</tbody>
</table>

$E$ – elastic modulus; $K$ – bulk modulus; $\mu$ – Poisson’s ratio; $\beta$ – relaxation coefficient; $G_0$ – maximum shear modulus; $G_1$ – equilibrium shear modulus.
behaviour of rubbers and mechanical rubber goods, for the purpose of finite element analysis in thermomechanical formulation, a viscoelastic model of material was proposed. In this work, the nature of the influence of the loading rate and temperature on the parameters of the model has been shown. On the basis of the results obtained, features of the determination and calculation of the elastic modulus ($E$), the damping factor during relaxation ($\beta$), the bulk modulus ($K$), and Poisson’s ratio ($\mu$) have been determined.

REFERENCES

Received 30.09.2013