Three-dimensional mathematical model of a single-screw plasticising extruder. 2: mathematical model for determining the screw temperature

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The quality of articles produced on extrusion equipment depends in many ways on the temperature regime under which they are processed. A controllable technological parameter is the temperature of the barrel. The screw temperature is established naturally during heat exchange and is dependent upon the prescribed temperature of the barrel, the number of revolutions of the screw, the consumption of the material, and its thermophysical and rheological properties. In this case, during operation of the extruder, nothing is known about the temperature distribution in the screw. Some screws have a longitudinal hole which is intended for its cooling. When the screw is cooled with water, the conditions of heat removal are chosen empirically.

The temperature distribution in the screw can be judged from experimental data or from the results of numerical investigation using the corresponding mathematical models. Since the screw rotates, while the length of the hole in the screw that is intended for its cooling is much greater than its diameter (roughly 100:1), the application of industrial devices to control the screw temperature is difficult. Therefore, the most promising method for determining the screw temperature and controlling it is the use of mathematical modelling.

To construct a model for determining the screw temperature, the following assumptions are introduced. The process is considered to be steady state. The screw channel (Figure 1) is replaced with an annular gap (Figure 2), with account taken of the ratio of the volumes of the screw thread and the screw channel. The complex movement of the polymer in the screw channel of the extruder is replaced with rod movement in a coaxial gap. The speed is determined from the mass flow rate. The thermophysical properties of steel and polymer are constant quantities that are independent of temperature.
On the basis of the simplifying assumptions made, the problem of determining the temperature in the screw in axisymmetrical formulation reduces to solving the equation of energy for the polymer

$$\rho_p c_p \frac{\partial T}{\partial z} = \lambda_p \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right] + Q$$

(1)

and the equation of heat conduction for the screw

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} = 0$$

(2)

where $T$ is the temperature, $V_z$ is the average speed of movement of the polymer in the annular gap of the model, $\rho_p$ is the density of the polymer, $c_p$ is the specific heat of the polymer, $\lambda_p$ is the thermal conductivity, and $Q$ is the specific power of the dissipative heat source as a result of the action of forces of viscous friction, the distribution of which over the length is determined by means of the model [1].

To obtain a complete mathematical formulation of the problem, equations (1) and (2) are supplemented with boundary conditions with respect to temperature.

The temperature of the barrel is determined by the processing regimes

$$T_{b} = T_0$$

(3)

If the screw has no hole, then, by virtue of the axisymmetry of the problem, on the axis the boundary condition has the form

$$\frac{\partial T}{\partial r} \bigg|_{r=0} = 0$$

(4)

If a screw with an axial longitudinal hole which can be used for cooling with water is examined (see Figure 2), then the following boundary condition of the third kind is used:

$$\frac{\partial T}{\partial r} \bigg|_{r=R_h} = -\frac{\alpha}{\lambda_m} (T_{b} - T_0)$$

(5)

where $\alpha$ is the coefficient of heat transfer, $T_0$ is the temperature of the moving medium in the hole, and $\lambda_m$ is the thermal conductivity of the steel screw.

The boundary condition at entry ($z = 0$) is set in the following way:

$$T_{z=0} = T_{w0}$$

$$T_{p} = T_p$$

(6)

where $T_{w0}$ is the screw temperature at $z = 0$, and $T_p$ is the temperature of the polymer at entry.

By virtue of the fact that the screw length is much greater than the screw diameter, the boundary condition at exit from the extruder ($z = L$) is determined from the condition of establishing a heat flow:

$$\frac{\partial T}{\partial z} = 0$$

(7)

At the metal–polymer interface, a boundary condition of the fourth kind should be fulfilled

$$\lambda_m \frac{\partial T}{\partial r} \bigg|_{r=R_h} = \lambda_p \frac{\partial T}{\partial r} \bigg|_{r=R_h}$$

(8)

The problem is solved by the finite element method.

The following procedure for calculating the screw temperature is proposed. The polymer performs rod motion in an annular gap between the screw and the internal surface of the cylinder, provided that ridges are absent, and here the heat flow from the barrel to the screw through the gap between ridges is taken into account iteratively. At each iteration step, the density of the heat flow from barrel to screw thread ridge through the radial gap is determined from the temperature difference between barrel and screw. The heat flow calculated in this way is applied to the screw. The iteration process will continue until the maximum difference in screw temperature between the current and the preceding iteration at nodes of the calculated region is lower in absolute magnitude than a certain prescribed small magnitude.

To check the adequacy of the mathematical model presented, the screw temperature of the ME-160 extruder was measured directly during the superposition of a second layer of polyethylene insulation on a KPB primary cable at the “Kamkabel” Works using a specially manufactured measuring device [2]. The solid points in Figure 3 show the results of experimental measurements, and the dashed line shows the change in the temperature of the barrel.

Figure 3. Change in temperature along axis
The thermal conductivity of the steel is equal to 50 W/(m K).

The thermophysical and rheological properties of high-density polyethylene of grade 271-74K are given in Table 1.

The screw temperature and other operating characteristics of the extruder are determined by a combined iteration procedure for solving the problem of heat and mass transfer of the polymer in the extruder channel [1] and the present problem of determining the screw temperature. At the first step, the screw temperature distribution is calculated on condition that there is no internal heat source. This distribution is used as a boundary condition in the mathematical model [1], after the solution of which the change in power of the internal heat source along the axis is determined (see Figure 5). Then, with account taken of the obtained distribution of dissipative heat source, the screw temperature is recalculated and the problem is solved again [1]. The procedure for combined solution of problems of heat and mass transfer of the polymer in the extruder channel [1] and determining the screw temperature will have to be continued until the prescribed condition of accuracy is fulfilled. As shown by the results of numerical investigations, two iteration procedures are quite sufficient in determining the screw temperature with an accuracy of up to 2%.

![Figure 4](image-url)  
**Figure 4.** Change in depth of screw channel

![Figure 5](image-url)  
**Figure 5.** Distribution of dissipative heat source along axis

The continuous line in Figure 3 shows the temperature calculated using the model proposed. From the figure it can be seen that there is fairly good agreement between the experimental data and the calculated curve. The maximum discrepancy between the experimental data and the results of calculations does not exceed 7%.

**Figure 6** shows the curves of change in the screw temperature along the axis, calculated: with account taken of the dissipative source and heat flows through the gap between the thread ridge and the barrel (curve 1); ignoring the dissipative source (curve 2); ignoring both factors (curve 3). From Figure 6 it can be seen that the influence of the given factors is considerable. The temperature difference on a significant length of the screw amounts to about 100 K, which can lead to considerable difficulties in determining the temperature of the materials and the pressure at exit from the extruder.

![Figure 6](image-url)  
**Figure 6.** Change in temperature along screw

<table>
<thead>
<tr>
<th>$\rho$, kg/m$^3$</th>
<th>$\lambda$, W/(m K)</th>
<th>$c$, J/(kg K)</th>
<th>$T_{m}$, °C</th>
<th>$\mu_0$, Pa s</th>
<th>$T_0$, °C</th>
<th>$\beta$, 1/K</th>
<th>$n$</th>
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<tr>
<td>850</td>
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<td>3150</td>
<td>117</td>
<td>25 000</td>
<td>200</td>
<td>0.007</td>
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</tbody>
</table>

Table 1. Properties of polyethylene of grade 271-74K
The model set out in the present work, together with the model in [1], makes it possible to obtain the most complete picture of the processing of polymers on single-screw extrusion units for different geometric and technological parameters, and also the properties of the material being processed. Furthermore, using this approach, it is possible to select the water cooling schedule of the screw in order to increase the throughput of the extruder or produce the required temperature regime of processing.

REFERENCES
