Prediction of Cushion Curves for Closed Cell Polyethylene-Based Foams. Part I. Modelling

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ABSTRACT

The possibility of prediction of the cushioning performance of closed cell foams under compressive impacts has been analyzed, using simple theoretical models. Several corrections have been introduced into the models previously proposed by Burgess. These corrections take into account the variation of gas volume in the foam with density, they incorporate the effect of the falling weight into the calculations, and finally they consider the base polymer effect. Even though the predictability of the model is not complete it provides a useful tool from the practical viewpoint.

1. INTRODUCTION

The variety of polymeric foams makes them suitable for a wide range of applications, in which their unique combination of properties allows redesign of several products such as packaging or helmet liners[(1-6)]. Moreover, properties such as light weight, strong colours, good ageing, chemical resistance and inertness, cushioning performance, thermal insulation, etc, have ensured that polyolefin-based foams have penetrated the automotive, building, marine, medical and packaging markets[(7-8)]. In many applications of these materials (packaging, personal protection against impact, etc) dynamic loads and impact can be expected[(9-11)].

Although many polymeric materials can be produced as foams, the most commonly used for packaging-cushioning purposes are polystyrene, polyethylene, polyurethane and some rubbers[(12)].

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The subject of the impact response of low-density foams has been reviewed in a general way by Mustin[12] and Mills[9], but the increasingly high range of polymer foams and processing methods available make necessary continuous research on this subject.

An optimum cushioning material needs to dissipate the kinetic energy of the impact while providing a gradual deceleration mechanism, keeping at the same time, the force on the protected item below some limit[8,13].

In polymer-based foams energy is absorbed as the struts buckle or cell walls bend or stretch viscoelastically, viscoplastically or plastically, depending on the mechanical properties of the base polymer[8,14]. When the foams are closed-celled, gas compression takes place providing pneumatic cushioning[9] as an additional energy absorption mechanism.

The accepted method to select the foams used in the packaging of fragile items uses cushion curves[9]. In these curves, the deceleration peak values, (e.g. the deceleration maxima of the striker during impact), are plotted as a function of the static stress for fixed falling height and foam thickness. In addition, foam manufactures publish quantitative data relative to shock absorbing properties in the form of cushion curves[15].

Several attempts have been made to obtain cushion curves from compressive stress-strain curves measured under static conditions[14-18]. However, when cushion curves are obtained from constant-rate measurements the mechanism of rapid gas compression is ignored. Moreover, all polymers are viscoelastic to some extent[19] and this means that the yield stress increases as the strain rate increases which results in different static and dynamic stress-strain relationships. Also, it is important to remark that even though for some base polymers (like polystyrene) stress-strain curves seem to follow a master curve; this is not the general case[19].

Some researchers obtained cushion curves from a limited series of impact tests[19-20] or from mechanical models[10-11]. In spite of partial successes, purely mechanical models are unable to predict adequately both curve values[21] and the asymptotic behaviour at low static stresses[19]. In addition, some authors proposed that thermodynamic phenomena play a significant role in the dynamic response of closed-cell foams[21-22]. As far as we know, the main contribution to the thermodynamic study of the impact response of the gas contained in closed-cell polymer based foams is due to Burgess[21]. In this paper, the cited author studied the behaviour of the gas contained in closed cell foams under dynamic loads, modelling it as an adiabatic or/and an isothermal process and obtaining the cushion curves equation for both cases as a function of parameters such as foam thickness, drop height, effective pressure of the gas and static stress.

The measurement of cushion curves according to either BS 4443 (section 3, Method 4) or ASTM D-1596-78a, requires an enormous amount of experimental work, sample preparation time and data analysis(19,21). For instance, to obtain the foam cushion curves for six sample thickness values, seven falling heights, ten different static stresses, five successive impacts and three repetitions, 6200 experiments are needed for just one foam density. Obviously, this is the best method to obtain design data but it is time consuming and expensive. Furthermore, in cushion design it is important to know the evolution of the minima of cushion curves with foam density, thickness and falling height. In order to obtain this information, it would be helpful to have analytical solutions for cushion curves in terms of the cited parameters.

The aim of this paper is to revise critically and improve the isothermal and adiabatic models proposed by Burgess including effects such as the foam density, the weight of the striker and the effect of the base polymer. The predictions of the models have been compared with the experimental behaviour of closed cell polyolefin foams.

The paper has two parts. In the first one, the models are presented and in the second one the experimental results are compared with the main models predictions.

2. BURGESS'S MODELS

Figure 1 shows several examples of cushion curves. The experimental values of the deceleration peaks G_{max} (maximum deceleration of the striker during the impact) were measured as a function of the static stress σ_s (ratio between the weight of the falling height and the surface area of the sample) in compressive impact tests (Figure 2) using a home-made impact test equipment, which has been previously described(23).

The results showed in Figure 1 correspond to a 40 kg/m^3 low density polyethylene closed cell foam of 50 mm thickness. It can be observed that deceleration values strongly depend on static stress and falling height (for a fixed sample thickness). Moreover, the trend as a function of the static stress also depends on falling height; increasing the falling height shifts the minimum of the curve to lower static stresses.

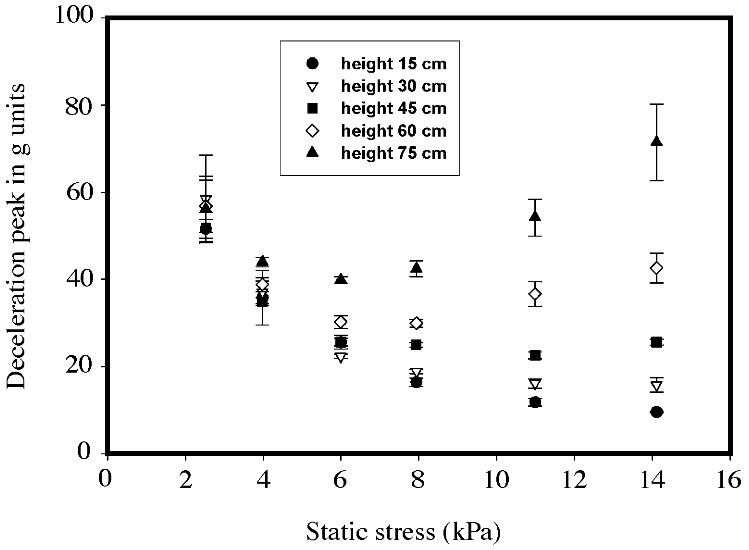

Figure 1. Cushion curves for a closed cell low density polyethylene foam in the first impact. Foam density and foam thickness were 40 kg/m^3 and 50 mm respectively

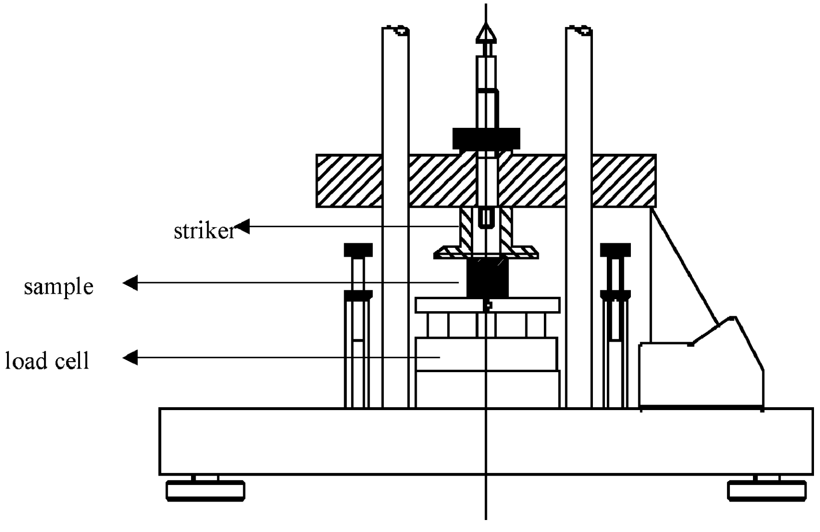

Figure 2. Lower part of the instrumented impact tester used in this investigation

The previous experimental behaviour which is characteristic of any low density closed cell foam is partly due to the compression of the gas. Therefore, the first attempt to model the foam performance is based on the analysis of the gas compression during an impact. Assuming that this compression is an adiabatic process, Burgess[(21)] obtained, for a given drop height (h), foam thickness (b) and static stress (σ_s), the following expression for cushion curves (in g units).

$$G_{\max} = \frac{P_0}{\sigma_s}\left(1+R\right)^{\frac{n}{n-1}} \quad (1)$$

where P_0 represents the effective pressure of the gas inside the cells, $n=c_p/c_v$ is the ratio of specific heats for the gas and R is a parameter that depends on static stress (σ_s), P_0, n, drop height (h) and foam thickness (b):

$$R = \frac{\sigma_s\left(n-1\right)h}{P_0 b} \quad (2)$$

If the gas compression is supposed to be isothermal[(21)] the cushion curves can be predicted by:

$$G_{\max} = \frac{P_0}{\sigma_s}\exp\left(\frac{\sigma_s h}{P_0 b}\right) \quad (3)$$

The previous relationships were obtained assuming several simplifications:

- The volume of the gas in the foam does not depend on density.
- The weight of the falling height was neglected
- The possible effect of the base polymer in the cushioning performance was not considered.

The functional dependency given by the previous models (equations 1 and 3) is very accurate. Figure 3 shows an example of the fitting of experimental results (for 40 kg/m^3 low density polyethylene foam being the falling height 45 cm and sample thickness 30 mm) to the isothermal and adiabatic models. The functions used in these fittings were:

$$G_{\max} = \frac{p_1}{\sigma_s}\left(1+p_2\sigma_s\right)^{p_3} \quad \text{(Adiabatic model)} \quad (4)$$

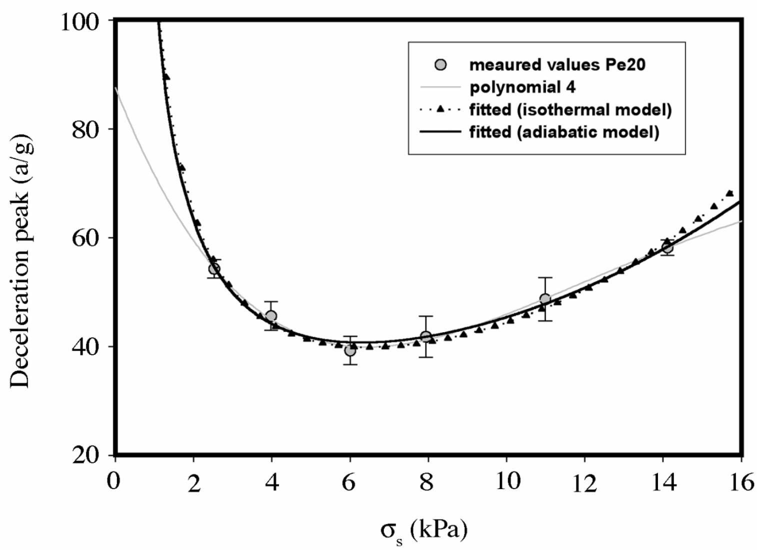

Figure 3. Fitting of the measured values for a polyethylene based foam (density 40 kg/m^3, sample thickness 30 mm and drop height 45 cm) to the G curves predicted by the isothermal and adiabatic models. A phenomenological fit to a grade 4 polynomial is also included to show how this fit is unable to predict the asymptotic part of these curves

$$G_{max} = \frac{p'_1}{\sigma_s} \exp(\sigma_s p'_2) \quad \text{(Isothermal model)} \qquad (5)$$

being p_1, p_2, p_3, p'_1 and p'_2 fitting parameters. Three and two free parameters were used in the adiabatic an isothermal models respectively.

In the same figure, it is also showed a polynomial fit with five free parameters (4th order). Nevertheless the five free parameters, the polynomial fit is unable to predict the asymptotic part of the curve.

As it has been proved, the two models proposed by Burgess can be used to fit the cushion curves. This interesting fact seems to be related to the key role played by the gas compression in the impact response of low density closed cell foams.

3. CORRECTIONS TO THE MODELS

a) Density Correction

Burgess considered that the ratio between the gas volume (V_{gas}) and the sample load bearing area (A) is the sample thickness (b).

$$\frac{V_{gas}}{A} = b \tag{6}$$

This is accurate if gas volume is equal to sample volume (very low density foams), but for foams of any density[8]:

$$V_{gas} \approx \left(1 - \rho_f / \rho_s\right) V_{sample} \tag{7}$$

where V_{gas} is the gas volume, V_{sample} is sample volume, ρ_f is the foam density and ρ_s is the base polymer density. Using the previous equation a first minor correction can be obtained.

Taking into account that the volume of gas in the foam depends on density:

$$\frac{V_{gas}\left(1 - \rho_f / \rho_s\right)}{A\left(1 - \rho_f / \rho_s\right)} \approx \frac{V_{sample}}{A}\left(1 - \rho_f / \rho_s\right) = b\left(1 - \rho_f / \rho_s\right) \tag{8}$$

It is possible to obtain the following expression for the adiabatic model:

$$G_{\max} = \frac{P_0}{\sigma_s}\left(1 + R\right)^{\frac{n}{n-1}} \tag{9}$$

where R is:

$$R = \frac{\sigma_s\left(n-1\right)h}{P_0 b\left(1 - \rho_f / \rho_s\right)} \tag{10}$$

The final result for the isothermal model is:

$$G_{\max} = \frac{P_0}{\sigma_s}\exp\left[\frac{\sigma_s h}{P_0 b\left(1 - \rho_f / \rho_s\right)}\right] \tag{11}$$

b) Striker Weight Correction

A second, more important correction to the Burgess's models, is related to the weight of the striker. In the impact tests showed in Figure 1 the maximum striker weight was 70.96 N, which is 0.3 times smaller than the result of multiplying the gas pressure and load sample area (supposing that gas pressure is atmospheric). Therefore, it is not clear that the effect of the striker weight can be neglected. Due to this reason, we have revised the theory, including this effect. The correction has been applied to the adiabatic case; a similar method could be used for the isothermal model. The calculations are detailed in the appendix, the final result for the deceleration peak is:

$$G_{\max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}} - 1 \tag{12}$$

where R is a parameter that depends on static stress (σ_s), P_0, n, drop height (h), strain maxima (ε_{max}), the ratio of foam density to polymer density (ρ_f/ρ_s) and foam thickness (b):

$$R = \frac{(h + b\varepsilon_{\max})(n-1)\sigma_s}{P_0 b\left(1 - \rho_f/\rho_s\right)} \tag{13}$$

c) Base Polymer Effect

A natural way to improve the Burgess's models is to consider the effect of the base polymer.

In our calculations it is assumed that the dynamic stress ($\sigma_{dynamic}$) can be separated into two terms, the first one related to gas pressure (P) and the second one to the stress beared by the polymeric structure ($\sigma_{polymer}$):

$$\frac{F}{A} = \sigma_{dynamic} = P + \sigma_{polymer} \tag{14}$$

where F is the contact force between the striker and the sample, and A is the sample area. A second approximation is that $\sigma_{polymer}$ is considered as a constant value as a function of strain, which corresponds to the yield stress of the foam under analysis. Considering the gas compression as an adiabatic process, it is possible to obtain (Appendix):

$$G_{\max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}} - 1 \tag{15}$$

where R is given by:

$$R = \frac{\sigma_s(n-1)}{P_0(1-\rho_f/\rho_s)}\left[\frac{h}{b}+\varepsilon_m\left(1-\frac{\sigma_y}{\sigma_s}\right)\right] \qquad (16)$$

Table 1 summarises the different successive approximations and the equations obtained in each model

Figure 4 shows the predicted values for the different adiabatic models and the measured data for a given foam. The values of the parameters used to compute these curves were P_0=101 kPa, n=1.4, σ_y=85 kPa and ρ_f=40 kg/m^3 (σ_y was experimentally measured from stress-strain curves in impact conditions). The predictions of the adiabatic model due to Burgess, adiabatic model+minor density correction, and adiabatic model+striker weight correction are very similar between them and are close to the experimental values for high values of the static stress.

The adiabatic model+base polymer effect seems to predict more accurately the foam behaviour in the range of low and medium static stresses, indicating

Table 1. Different successive approximations and the equations obtained in each model

Model	Cushion curves equation	Constants
Adiabatic	$G_{max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}}$	$R = \frac{\sigma_s(n-1)h}{P_0 b}$
Adiabatic with density correction	$G_{max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}}$	$R = \frac{\sigma_s(n-1)h}{P_0 b(1-\rho/\rho_s)}$
Adiabatic with weight correction	$G_{max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}} - 1$	$R = \frac{(h+b\varepsilon_{max})(n-1)\sigma_s}{P_0 b(1-\rho/\rho_s)}$
Isothermal	$G_{max} = \frac{P_0}{\sigma_s}\exp\left(\frac{\sigma_s h}{P_0 b}\right)$	-
Isothermal with density correction	$G_{max} = \frac{P_0}{\sigma_s}\exp\left[\frac{\sigma_s h}{P_0 b(1-\rho/\rho_s)}\right]$	-
Adiabatic + Polymer	$G_{max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}} - 1$	$R = \frac{\sigma_s(n-1)}{P_0(1-\rho_f/\rho_s)}\left[\frac{h}{b}+\varepsilon_m\left(1-\frac{\sigma_y}{\sigma_s}\right)\right]$

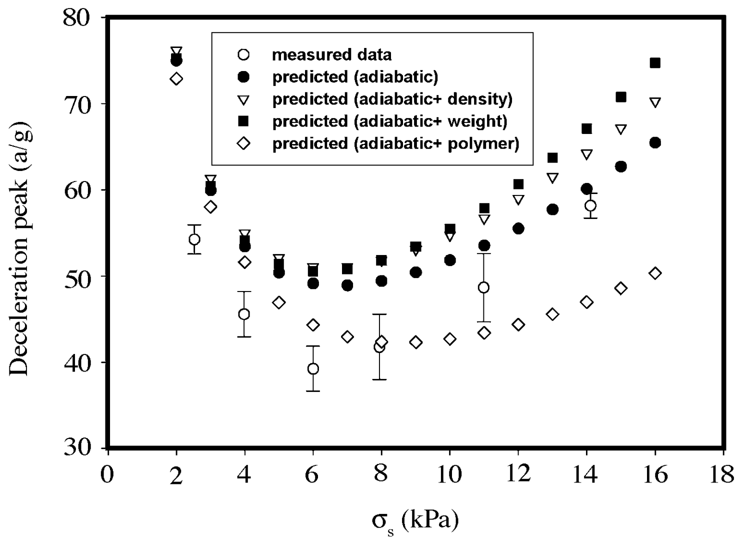

Figure 4. Measured cushion curves for a polyethylene based foam (Pe20 with density 39.9 kg/m^3, sample thickness 30 mm and drop height 45 cm) compared to the predictions of the adiabatic model due to Burgess, adiabatic model + minor density correction, adiabatic model+ striker weight correction and adiabatic plus base polymer effect

that it seems to be necessary to consider the foam structure deformation as an energy absorbing mechanism.

A key aspect of the models is their capability to predict the evolution of the minima of the cushion curves as a function of the experimental testing conditions (drop height, foam thickness and static stress) for a fixed foam density, and the agreement of these predictions with experimental curves. An analysis of this problem will be presented in the second part of this paper.

CONCLUSIONS

It has been shown that a minor correction can be added to the Burgess's model for the analytical expressions of cushion curves to take into account that the amount of gas contained inside a closed cell foam depends on density. A second parallel calculation has been presented to incorporate the effect of the striker weight into the models. Finally, the base polymer effect has been introduced to obtain an expression for the cushion curves that depends on static stress, the ratio of foam thickness to falling height, foam relative density, initial pressure of the gas inside the cells, ratio of specific heats of the gas, maximum strain and foam yield stress.

Burgess's analysis does not provide a truly satisfactory method to predict the cushion curves, but it has been found to be useful in the fitting of the experimental values of these curves. In particular this approach can be used as a technique for modelling.

APPENDIX

Striker Weight Correction:

The force balance requires (Figure A.1):

$$PA - W = \left(\frac{W}{g}\right)a_{cc} \qquad \text{A.1}$$

Where P stands for the pressure of the gas, W for the striker weight and a_{cc} is the acceleration of the striker.

Assuming non lateral expansion and a polytropic process:

$$a_{cc} = \frac{1}{A}\frac{d^2V}{dt^2} \qquad \text{A.2}$$

$$P_0V_0^n = PV^n \qquad \text{A.3}$$

where V_0 and V are the gas volumes in two different times during an impact:

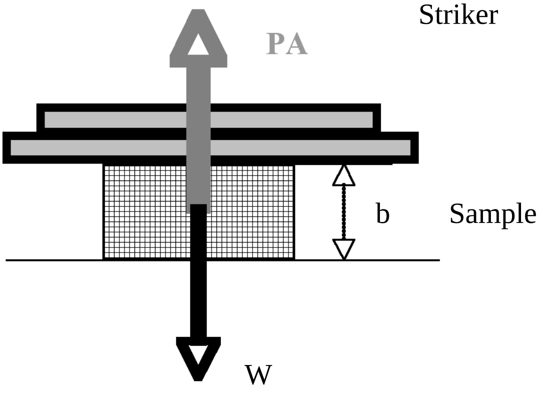

Figure A.1. Scheme of forces acting during an impact: W is the striker weight and PA is the result of multiplying the gas pressure and load sample area

$$PA = W\left(\frac{a_{cc}}{g}+1\right) \Rightarrow \left(\frac{PA}{W}-1\right)g = a_{cc} \qquad \text{A.4}$$

Then

$$\frac{d^2V}{dt^2} = \left[\frac{P_0A}{W}\left(\frac{V_0}{V}\right)^n - 1\right]Ag \qquad \text{A.5}$$

The initial conditions are:

$$t = 0 \Rightarrow V = V_0 \qquad \text{A.6}$$

$$t = 0 \Rightarrow \frac{dV}{dt} = -A\sqrt{2gh} \qquad \text{A.7}$$

Integrating both sides of (A.5) and taking into account that:

$$\int_{V_0}^{V} \frac{d^2V}{dt^2}dV = \int_{V_0}^{V} d\left[\frac{1}{2}\left(\frac{dV}{dt}\right)^2\right] = \left[\frac{1}{2}\left(\frac{dV}{dt}\right)^2\right]_{V_0}^{V} = \frac{1}{2}\left(\frac{dV}{dt}\right)^2 - \frac{A^2}{2}(2gh) \qquad \text{A.8}$$

It is obtained:

$$\frac{1}{2}\left(\frac{dV}{dt}\right)^2 - A^2gh = \frac{P_0AV_0}{(1-n)W}\left[\left(\frac{V_0}{V}\right)^{n-1} - 1\right]Ag - Ag\left[V - V_0\right] \qquad \text{A.9}$$

Assuming that the maximum deceleration occurs when the minimum volume is reached:

$$\frac{dV}{dt} = 0 \Rightarrow V = V_m \qquad \text{A.10}$$

Then:

$$-Ah = \frac{P_0V_0}{(1-n)\sigma_s}\left[\left(\frac{V_0}{V_m}\right)^{n-1} - 1\right] - \left[V_m - V_0\right] \qquad \text{A.11}$$

where V_o and V_m are the initial and minimum gas volumes and

$$V_m \approx Ab\left(1-\varepsilon_{\max}\right)\left(1-\rho_f/\rho_s\right) \qquad \text{A.12}$$

Hence:

$$-h \approx \frac{P_0 b\left(1-\rho_f/\rho_s\right)}{\left(1-n\right)\sigma_s}\left[\left(\frac{V_0}{V_m}\right)^{n-1}-1\right]+b\varepsilon_{\max} \qquad \text{A.13}$$

Then, the relationship between the initial and minimum gas volumes during the impact can be obtained from:

$$\left(\frac{V_0}{V_m}\right)=\left\{1+\frac{\left(h+b\varepsilon_{\max}\right)\left(n-1\right)\sigma_s}{P_0 b\left(1-\rho_f/\rho_s\right)}\right\}^{\frac{1}{n-1}} \qquad \text{A.14}$$

Defining R as: $\left(\frac{V_0}{V}\right)=\left\{1+R\right\}^{\frac{1}{n-1}}$ A.15

Therefore, the following expression can be deduced:

$$\left(\frac{V_0}{V}\right)=\left\{1+R\right\}^{\frac{1}{n-1}} \qquad \text{A.16}$$

However, for a general polytropic process the relationship between pressure and volume follows:

$$P_{\max}=P_0\left(\frac{V_0}{V_m}\right)^n \qquad \text{A.17}$$

where P_{max} is the maximum pressure inside the cells. Assuming that this value corresponds to the minimum gas volume:

$$P_{\max}=P_0\left(1+R\right)^{\frac{n}{n-1}} \qquad \text{A.18}$$

Using the force balance equation:

$$PA-W=\left(\frac{W}{g}\right)a_{cc} \qquad \text{A.19}$$

Calling $G = a_{cc}/g$, to the deceleration of the striker in g units, and supposing that the maximum deceleration corresponds to the maximum pressure inside the cells (to the minimum gas volume), $G_{\max} = (a_{cc})_{\max}/g$, the cushion curve equation can be obtained from:

$$G_{\max} = \frac{(a_{cc})_{\max}}{g} = \frac{P_{\max} A}{W} - 1 = \frac{P_{\max}}{\sigma_s} - 1 \qquad \text{A.20}$$

Taking into account the P_{max} expression obtained previously:

$$G_{\max} = \frac{P_0}{\sigma_s}(1+R)^{\frac{n}{n-1}} - 1 \qquad \text{A.21}$$

$$R = \frac{(h + b\varepsilon_{\max})(n-1)\sigma_s}{P_0 b(1 - \rho_f/\rho_s)} \qquad \text{A.22}$$

Base Polymer Effect

The theory starts once again from the force balance equation:

$$F - W = \left(\frac{W}{g}\right) a_{cc} \qquad \text{A.23}$$

Dividing the equation by A, the load bearing sample area:

$$\frac{F}{A} - \frac{W}{A} = \left(\frac{W}{Ag}\right) a_{cc} \Rightarrow \frac{F}{A} - \sigma_s = \left(\frac{\sigma_s}{g}\right) a_{cc} \qquad \text{A.24}$$

As a first approximation:

$$\frac{F}{A} = \sigma_{dynamic} = P + \sigma_{polymer} \qquad \text{A.25}$$

where P is the gas pressure and $\sigma_{polymer}$ is the stress supported by the polymeric structure.

The deceleration of the striker can be obtained from:

$$a_{cc} = \frac{1}{A}\frac{d^2V}{dt^2} \qquad \text{A.26}$$

Then:

$$P + \sigma_{polymer} - \sigma_s = \frac{\sigma_s}{Ag}\frac{d^2V}{dt^2} \qquad \text{A.27}$$

Integrating both sides of this equation:

$$\int_{V_0}^{V_m} PdV + \int_{V_0}^{V_m} \sigma_{polymer} dV - \int_{V_0}^{V_m} \sigma_s dV = \int_{V_0}^{V_m} \frac{\sigma_s}{Ag}\frac{d^2V}{dt^2} dV \qquad \text{A.28}$$

Integrating each term:

First term:

$$\int_{V_0}^{V_m} PdV_{probeta} = \frac{P_0 V_0}{1-n}\left[\left(\frac{V_0}{V_m}\right)^{n-1} - 1\right] \qquad \text{A.29}$$

Second term:

$$\int_{V_0}^{V_m} \sigma_{polymer} dV \qquad \text{A.30}$$

If $\sigma_{polymer}$ is considered as a constant value, which corresponds to the yield stress (σ_y) of the foam:

$$\int_{V_0}^{V_m} \sigma_{polymer} dV = \sigma_y Ab\left(-\varepsilon_m\right) \qquad \text{A.31}$$

Third term:

$$\int_{V_0}^{V_m} \sigma_s dV = \sigma_s Ab\left(-\varepsilon_m\right) \qquad \text{A.32}$$

Fourth term:

$$\int_{V_0}^{V_m} \frac{\sigma_s}{Ag}\frac{d^2V}{dt^2} dV = -\frac{\sigma_s}{2g} Ab^2\varepsilon_0^2 = -\sigma_s Ah \qquad \text{A.33}$$

Introducing these results into the original equation:

$$\frac{P_0V_0}{1-n}\left[\left(\frac{V_0}{V_m}\right)^{n-1}-1\right]-\sigma_y Ab\varepsilon_m+\sigma_s Ab\varepsilon_m=-\sigma_s Ah \qquad \text{A.34}$$

It can be obtained that:

$$\left(\frac{V_0}{V_m}\right)^{n-1}=1-\frac{\sigma_s h(1-n)}{P_0 b(1-\rho_f/\rho_s)}-\frac{\sigma_s\varepsilon_m(1-n)}{P_0(1-\rho_f/\rho_s)}+\frac{\sigma_y Ab\varepsilon_m(1-n)}{P_0V_0} \qquad \text{A.35}$$

Then:

$$\left(\frac{V_0}{V_m}\right)^{n-1}=1+\frac{\sigma_s(n-1)}{P_0(1-\rho_f/\rho_s)}\left[\frac{h}{b}+\varepsilon_m\left(1-\frac{\sigma_y}{\sigma_s}\right)\right] \qquad \text{A.36}$$

Defining R as:

$$R=\frac{\sigma_s(n-1)}{P_0(1-\rho_f/\rho_s)}\left[\frac{h}{b}+\varepsilon_m\left(1-\frac{\sigma_y}{\sigma_s}\right)\right] \qquad \text{A.37}$$

We can reach the following equation:

$$\left(\frac{V_0}{V_m}\right)=(1+R)^{\frac{1}{n-1}} \qquad \text{A.38}$$

Where V_0, V_m are the initial and minimum gas volumes during the impact.

It has been obtained similar equations to the ones:

$$P_{\max}=P_0(1+R)^{\frac{n}{n-1}} \qquad \text{A.39}$$

$$T_{\max}=T_0(1+R) \qquad \text{A.40}$$

From $G=\frac{a_{cc}}{g}$, it is deduced that for the maximum values $G_{\max}=\frac{(a_{cc})_{\max}}{g}$, which correspond to $P_{\max}$:

$$G_{\max}=\frac{(a_{cc})_{\max}}{g}=\frac{P_{\max}A}{W}-1=\frac{P_{\max}}{\sigma_s}-1 \qquad \text{A.41}$$

Introducing $P_{\max} = P_0 \left(\frac{V_0}{V_m} \right)^n$ and from (A.36):

$$G_{\max} = \frac{P_0}{\sigma_s} (1+R)^{\frac{n}{n-1}} - 1 \qquad \text{A.42}$$

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